FE | Sem. I (Rev.) A.M. I

4/12/12

P4-RT-Exam-Oct-12-150

Con. 8962-12. (REVISED COURSE) KR-3357

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **three** questions from the remaining **five**.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Prove that $\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{1-\cosh^2 x}}} = \cosh^2 x$ **3**

(b) If $u = \log[\tan x + \tan y]$, prove that, **3**

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2.$$

(c) If $u = \dfrac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$ **3**

(d) Show that $\log[1+\sin x] = x - \dfrac{x^2}{2} + \dfrac{x^3}{6} + \ldots\ldots\ldots\ldots$ **3**

(e) Show that every square matrix can be uniquely expressed as P + iQ, where P and Q are Hermitian matrices. **4**

(f) Find n^{th} order derivative of $\dfrac{x^2+4}{(x-1)^2(2x+3)}$ **4**

2. (a) Show that the roots of the equation $(x+1)^6 + (x-1)^6 = 0$ are given by **6**

$$-i\cot\left[\frac{(2k+1)\pi}{12}\right],\ k = 0, 1, 2, 3, 4, 5.$$

(b) Reduce the following matrix into normal form and find its rank **6**

$$\begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$$

(c) State and prove the Euler's theorem for a homogeneous function in two variables. **8**

Hence find the value of $x \cdot \dfrac{\partial u}{\partial x} + y \cdot \dfrac{\partial u}{\partial y}$ if $u = \dfrac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$.

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3. (a) Test for consistency and solve if consistent – 6

$$x_1 - 2x_2 + x_3 - x_4 = 2\ ;\ x_1 + 2x_2 + 2x_4 = 1\ ;$$
$$4x_2 - x_3 + 3x_4 = -1$$

(b) Find all the stationary values of $x^3 + 3xy - 15x^2 - 15y^2 + 72x$. 6

(c) If $\tan\left(\frac{\pi}{4} + iv\right) = re^{i\theta}$, show that 8

(i) $r = 1$ (ii) $\tan\theta = \sinh 2v$ (iii) $\tanh v = \tan\frac{\theta}{2}$.

4. (a) If $x = u + e^{-v}\sin u$, $y = v + e^{-u}\cos u$ find $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$ by using Jacobian. 6

(b) Considering only the principal value, if $(1 + i\tan\alpha)^{1+i\tan\beta}$ is real, prove that its value is $(\sec\alpha)^{\sec^2\beta}$. 6

(c) Solve the system of linear equation by Crout's method 8

$$x - y + 2z = 2\ ;\quad 3x + 2y - 3z = 2\ ;\quad 4x - 4y + 2z = 2.$$

5. (a) Expand $\cos^7\theta$ in a series of cosines of multiples of θ. 6

(b) Evaluate $\lim_{x\to 0}\left[\frac{1}{x^2} - \cot^2 x\right]$ 6

(c) If $y = \left(\sin^{-1}x\right)^2$, obtain $y_n(0)$. 8

6. (a) Show that the vectors are linearly dependent and find the relation between them :– 6

$$X_1 = [1, 2, -1, 0],\ X_2 = [1, 3, 1, 2],\ X_3 = [4, 2, 1, 0],\ X_4 = [6, 1, 0, 1]$$

(b) If $\frac{x^2}{1+u} + \frac{y^2}{2+u} + \frac{z^2}{3+u} = 1$, 6

Prove that,

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right]$$

(c) Fit a second degree parabolic curve to the following data :– 8

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9